CALIBRATION OF HUBBLE SPACE TELESCOPE FOCAL-LENGTH VARIATIONS USING THE EMBEDDING TECHNIQUE

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ABSTRACT

A new model has been generated to calibrate Hubble Space Telescope (HST) focal-length changes on a 5-minute time grid for the 1995-1999 period. This CHAOS Program Model (CPM) has been built by a software package, CHAOS, developed by Agilent Laboratories (a spinoff of Hewlett-Packard Laboratories). CHAOS uses the modern embedding technique developed initially to predict the behavior of nonlinear dynamic systems. The CPM fits the measured focus points about 36 percent better than the Full-Temperature Model (FTM), the best of the existing models. The CPM uses the same temperature functions as the FTM, minus four temperatures from sensors in the light shield near the secondary mirror. As demonstrated by this HST focal-length calibration, the embedding technique used by the CHAOS package has potential for application to a wide variety of HST/Next Generation Space Telescope (NGST) calibration problems.

INTRODUCTION

This paper describes the CHAOS Program Model (CPM) of Hubble Space Telescope (HST) focus behavior at 5-minute intervals for the period 1995-1999. It contains no information about the HST telescope framework and optical telescope assembly as related to thermal effects on the telescope focal length. A comprehensive description of these systems and thermal effects can be found in Space Telescope Science Institute (STScI) document SESD-97-01, which also describes three physical models – the Four-Temperature Model, the Attitude Model, and the Full-Temperature Model – created by the STScI's John Hershey. We do not discuss these models here, nor do we discuss their comparative pros and cons with respect to the CPM from the physical point of view. We do, however, discuss the comparative fits of the various models to the focus measurement data, and we compare model focus estimates at times between focus measurements.

We have used the CHAOS software package (CP) to develop the CPM. The **CHAOS Program** section of this paper provides a brief description of what CP is and how it works.[1]

[1] Agilent Labs developed the CP and some of its computational methods.

One goal of this paper is to demonstrate how the CHAOS Program can be used to solve spacecraft sensor calibration problems. We selected the HST focus calibration task for this initial demonstration project because of easy access to focus data and the existence of other focus models developed at STScI.[2] It is our belief that because of its flexibility in transfer function design, CP could offer significant savings over more traditional, custom-built calibration systems.

First of all, we have found that across the period from 1995 through early 1999, the CPM fits the observed focus data better than the existing Four-Temperature, Attitude, and Full-Temperature models. The root-mean-square (RMS) residuals for the CPM show 42 percent, 48 percent, and 36 percent improvements over the residuals for the three existing models (see details below).

Figure 1a shows focus observations along with FTM and CPM focus estimates from 1995 through the first quarter of 1997 [mostly before HST servicing mission 2 (SM2)]. Figure 1b plots the same data from the second quarter of 1997 through the first quarter of 1999. These two figures demonstrate how well the FTM and the CPM predict the observations. The **CHAOS Program Model** section of this paper provides additional details about these files.

CHAOS PROGRAM

The CP uses modern embedding prediction methods to enhance the traditional linear least-squares concept of loss function minimization and transfer function definition. The essence of the embedding technique can be seen from the following example. Consider two time series, or *state variables*. The first, f_{drive}, is the *driving* sequence, and the second is the *response* sequence, $f_{response}$ (i.e., the reaction of a system to the driving sequence). The system is considered to be a *black box*.

It is impossible to predict the instantaneous response to the drive if only the current value of the drive is used. As shown in Figure 2, the same drive value C causes two different responses: A and B. The key idea of the embedding technique is to use for the prediction not only the instant value of the drive signal, but also its values in times before the prediction. The first, second, and higher order time derivatives are good examples of embedding, because calculating the derivatives requires using values of signal in sequential time points. As is obvious from Figure 2, using time derivatives of the drive signal as additional data for the predictor can solve the problem of predicting values for points A and B, because the first time derivatives have different values for these points. Here, the embedding is an operation that transforms the initial set of data into a new (usually expanded) set of data. In Equation (1), the phase space of the system is expanded

[2] John Hershey of the STScI developed the existing focus models. An HST web site presents the focus data for these models at 5-minute intervals for each quarter of each year from 1994 through 1999. John Hershey provided us with raw temperature data for the same time period.

1995, 1996 AND 1997 (JANUARY, FEBRUARY, MARCH)

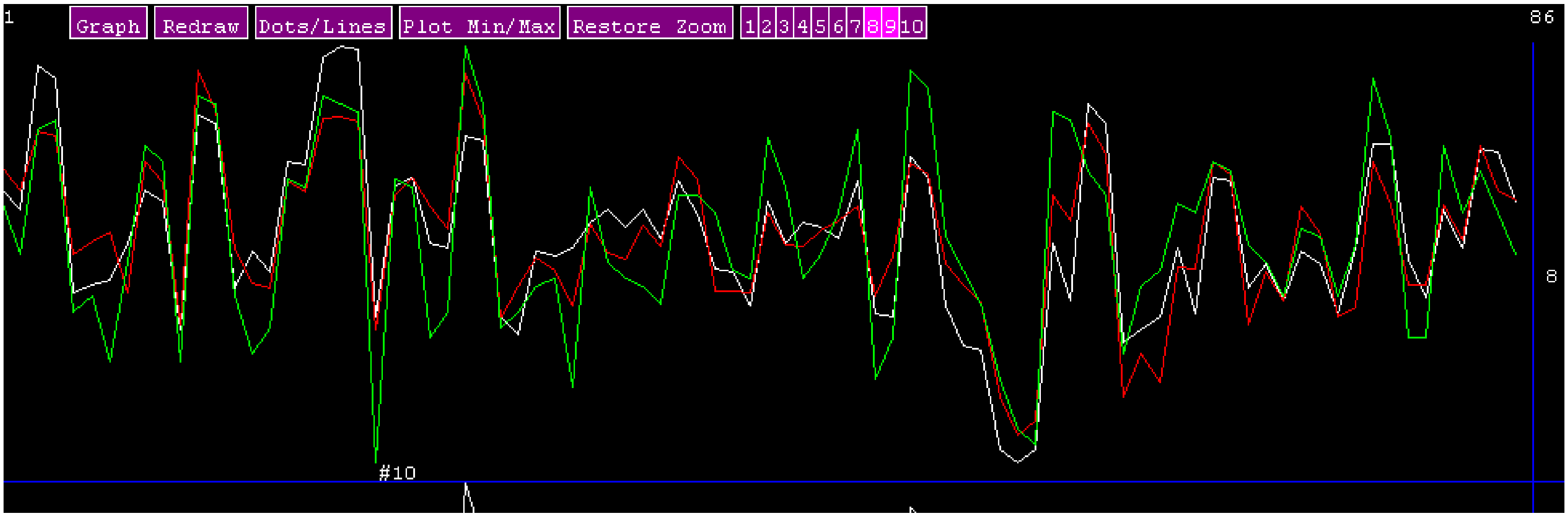
98 FOCUS OBSERVATIONS

WHITE: OBSERVATIONS
GREEN: FULL TEMPERATURE MODEL PREDICTIONS (HST) (STANDARD DEVIATION=1.040)
RED: CHAOS PREDICTIONS (STANDARD DEVIATION=0.735)

CHAOS PREDICTION IS 29.3% BETTER THAN FTM

(FTM IS FULL TEMPERATURE MODEL)

Figure 1a. Focus Observations and FTM and CPM Focus Estimates From 1995 Through First Quarter of 1997 (Mostly Before SM2)



1997 (APRIL THROUGH DECEMBER), 1998 AND 1999 (JAN, FEB., MARCH) (86 OBSERVATIONS)

WHITE: FOCUS OBSERVATIONS
GREEN: FULL TEMPERATURE MODEL (HST) PREDICTIONS (STANDARD DEVIATION=1.346)
RED: CHAOS PREDICTIONS (STANDARD DEVIATION=0.861)

CHAOS PREDICTION IS 36.0% BETTER THAN FTM

Figure 1b. Focus Observations and FTM and CPM Focus Estimates From Second Quarter of 1997 Through First Quarter of 1999 (Mostly After SM2)

from one-dimensional, which includes f_{drive} only, to two-dimensional, which includes the time derivatives of f_{drive} as well.

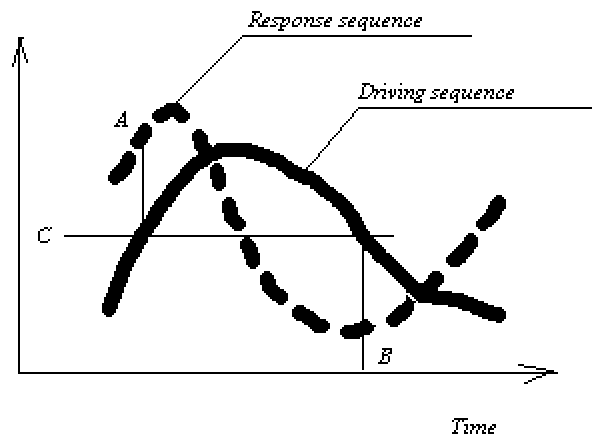


Figure 2. Embedding: Effect of Driving Sequence on Response Sequence

Often, even for nonlinear systems, a robust fit is likely if one uses a proper polynomial, as in the following transfer function:

$$f_{response} = x_0 + x_1 f_{drive} + x_2 \dot{f}_{drive} + x_3 f_{drive} \dot{f}_{drive} + x_4 \left(\dot{f}_{drive}\right)^2 + ... \ , \quad (1)$$

where $\dot{f}$ is the time derivative. The unknown polynomial coefficients $(x_0, x_1, x_2, x_3, x_4, ...)$ can be found by the least-squares method (LSM).[3]

The nonlinear model extraction is based on dynamic-reconstruction theory, which has its origins in the Takens embedding theorem and has been extensively developed for modeling autonomous systems by the scientific community over the past decade. The embedding theorem has a powerful implication: the evolution of points in the "reconstructed" state space follows that of the unknown dynamic in the original state space. Thus, the embedding theorem opens the way toward a general solution for extracting behavioral models for nonlinear devices directly from time-domain measurements.

We should recall that a model's accuracy is restricted by the amount of noise in the source data. If a process includes different internal time scales, then restrictions related to

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[3]The problem solved is a construction of a black box behavioral model for an object with nonlinear behavior (a polynomial of degree higher than one) describable by a functional or differential relation among the state variables. By *behavioral model*, we mean a set of parameters that define the input/output behavior of the object under investigation. In practical calculations, the behavioral model must be of a form suitable for rapid simulation. By *black box*, we mean a model that requires no physical information about the device before model building.

finite data accuracy can be partially avoided by a proper smoothing, such as that provided by the wavelet function. Wavelet smoothing can be explained using the following example. Consider a wavelet with width = 3. The wavelet value derived from a source variable is calculated on six sequential points in time: $I_6 = f_6 + f_5 + f_4 - f_3 - f_2 - f_1$. It is a kind of averaged first derivative (i.e., the derivative, which is smoothed over a span of points).

CHAOS PROGRAM MODEL

To build a model, we could use any of the data used to create the three already existing models. The first of these, the Four-Temperature Model, was derived early in the life of HST by Pierre Bely of the STScI's Science and Engineering Systems Division (SESD) and is based on four temperature sensors in the light shield near the secondary mirror spider. The second model, the Attitude Model, is based on HST attitude information from the mission scheduler files (Sun angles, off-nominal rolls, occultations, and day/night satellite positions). The third model, the Full-Temperature Model (FTM), is based on a large number of temperature sensors throughout the telescope, including the four sensors in the first model. Because the FTM represents the observations somewhat better than the other two models do, we chose to use the raw data used to create the FTM to build the CPM.

Table 1 shows an extraction from the typical raw data file (from the first quarter of 1997) used to build the CPM.

Table 1. Sample From Raw Data File

Year	Day	Hour	Min.	Modified Julian Date	Temperature Functions (degrees Celsius)						Observed Focus Position (microns)
					T1	T2	T3	T4	T5	T6	
1997	9	22	20	50457.92969	28.8	-9	-24.1	-8.4	-37.3	16.8	
1997	9	22	25	50457.93359	28.6	-8.8	-23	-8.6	-36	16.8	
1997	9	22	30	50457.93750	28.8	-9	-22.7	-8.5	-34	16.8	
1997	9	22	35	50457.94141	28.8	-9	-21.7	-8.1	-32.7	16.8	
1997	9	22	40	50457.94531	28.7	-9.2	-21.2	-8.1	-31.5	16.8	
1997	9	22	45	50457.94922	28.8	-9	-20	-7.9	-30.5	16.8	
1997	9	22	50	50457.95313	28.8	-9	-19.3	-7.8	-30.4	16.8	3.00E-01
1997	9	22	55	50457.95703	28.8	-9	-19.1	-7.6	-30.6	16.7	9.00E-01
1997	9	23	0	50457.96094	28.7	-9.2	-19	-7.4	-30.9	16.8	
1997	9	23	5	50457.96094	28.7	-9	-19.7	-7.4	-31.5	16.8	
1997	9	23	10	50457.96484	28.8	-9	-20.5	-7.4	-32.5	16.8	

T1, T2, T3, T4, T5, and T6 are six functions of many telescope temperature sensors, including four light shield temperatures near the secondary mirror; each function is the mean of a number of sensor measurements extracted by the Observatory Monitoring Program (OMS):

- T1: Truss axial temperature difference function
- T2: Truss diametrical temperature difference function
- T3: Aft shroud temperature function
- T4: Forward shield temperature function
- T5: Light shield temperature function
- T6: Primary mirror temperature function

The last column of the table contains focus position observations; in most cases, the value shown in this column is blank, indicating that no observations were made for the corresponding time row. The focus observations are measured in units of secondary mirror microns and are relative to the best focus (zero microns) of the wide field planetary camera 2 (WFPC2).[4]

We have excluded from consideration the focus measurements of 1994 and four focus observations of 1995 and 1996 because no regular temperature data (T1-T6) exist for the appropriate times. Consequently, we are dealing with a typical calibration task: using existing focus observations and appropriate raw data (T1-T6) for the time of observations to predict focus values for time rows with no observations. To do that, we must create the model of focus behavior using the existing set of observations.

We have divided the focus observations into two data sets. The first data set, mostly before SM2, includes the time period from 1995 to the first quarter of 1997 and contains 98 observations. The second data set, after SM2, includes the time period from the second quarter of 1997 to the first quarter of 1999 and contains 86 observations.

By experimenting, we have found one transfer function (embedding scheme) that applies equally well to both data sets. It uses the following terms: a constant offset; modified Julian date; temperature functions T1, T4, and T5; three-point wavelets of T1, T4, and T5; and the first derivative of T5:

$$F_E = C_0 + C_1 t + C_2 T1 + C_3 W(T1) + C_4 T4 + C_5 W(T4) + C_6 T5 + C_7 W(T5) + C_8 \frac{\Delta T_5}{\Delta t}$$

[4]According to convention (see http://www.stsci.edu/instruments/observatory/focus/focus2.html), we express all focus changes in the science instrument-independent units of secondary mirror microns. A +1-micron change is equivalent to a physical movement of the secondary mirror by that amount away from the primary mirror. One micron of secondary mirror defocus translates to 110 microns at the focal plane. The focus zero point is defined as WFPC2 best focus as determined by point spread function (PSF)-fitting (phase retrieval) software for PSFs in the ~400-800 nanometer range.

where

$$
\begin{aligned}
F_E &= \text{focus estimate (microns)} \\
t &= \text{modified julian date} \\
W(T) &= \text{three - point wavelet function of temperature function T} \\
\frac{\Delta T5}{\Delta t} &= \text{first derivative approximation for } T_5 \\
&= (T5(t_2) - T5(t_1))/(t_2 - t_1) \\
C_1 \cdots C_9 &= \text{polynomial coefficients}
\end{aligned}
$$

Calculations show that including T2, T3, and T6 in the transfer function does not improve the fit.

Note that the sensitivity to three-point wavelets means that the current value of the focus depends on the nearest time history of temperatures before the focus observation. The three-point wavelet uses six points of data; for a 5-minute time grid, this equals 25 minutes, about one-fourth of the orbital period for HST. The temperature inertia of sensors might be responsible for this result.

We have computed two sets of transfer function coefficients (one for data set one and one for data set two), and we have used these to evaluate the fit to the observed focus measurements. Table 2 shows standard deviations from focus observations for the Four-Temperature, Attitude, Full-Temperature, and CHAOS Program Models. In the first column, the notations 95_1, 97_2, etc., mean first quarter of 1995, second quarter of 1997, etc. One can see from Table 2 that the CPM fits the observed focus data points better than the existing Four-Temperature, Attitude, and Full-Temperature models.

Figures 3, 4, 5 and 6 show the FTM (green) and the CPM (red) fits to the observations (white). These figures illustrate the data in Table 2. They represent four quarter files of 1997, the year richest in observations. Figure 3 represents the first quarter of 1997 with 59 observations. Figure 4 represents the second quarter of 1997 with 16 observations. Figure 5 represents the third quarter of 1997 with 10 observations. Figure 6 represents the fourth quarter of 1997 with 8 observations. Thus, the number of observations (93) presented in these four files is more than half the total number of observations (184) used in this study.

Figure 7a gives an overview of data for the first quarter of 1997. It shows the behavior of all functions of interest with 5-minute time intervals. From this figure one can see the behavior of all three temperature functions (T1, T4 and T5), estimates of all four models, and observations. Figure 7b shows a fragment of the previous picture for a time interval of about 25-30 hours. Note the drop in the CPM focus estimates at a time when there is a jump in several of the temperature functions.

There are cases where the CPM shows larger amplitudes in its estimates than other models. An example can be found in Figure 8, which compares the behavior of the four models during the second quarter of 1998.

Table 2. Standard Deviations (microns)

Year_ Quarter	Four-Temp. Model	Attitude Model	Full-Temp. Model	CHAOS Program Model	Number Of Focus Observations	Comments
95_1	1.312	1.310	1.068	0.584	6	
95_2	2.2	2.2	2.2	0.56	1	
95_3	1.444	0.850	1.196	0.974	4	
95_4	1.116	1.113	0.775	0.693	6	One observation has been removed
96_1	0.995	1.603	0.789	0.780	3	One observation has been removed
96_2	0.913	1.294	0.469	0.446	3	Two observations have been removed
96_3	1.522	2.033	1.570	0.877	8	
96_4	1.917	1.354	1.577	1.015	8	
97_1	1.222	1.134	0.848	0.726	59	
97_2	1.186	1.268	1.078	0.919	16	
97_3	1.822	1.952	1.647	1.003	10	
97_4	0.925	0.753	0.870	0.664	8	
98_1	1.074	1.312	0.832	0.664	11	
98_2	1.917	2.765	1.967	0.940	18	
98_3	1.600	1.512	1.304	0.780	10	
98_4	0.685	1.251	0.699	0.844	10	
99_1	0.519	1.173	1.266	0.472	3	
Average Standard Deviation:	1.316	1.463	1.186	0.761	184	

97_1 (59 OBSERVATIONS)

GREEN: STANDARD DEVIATION=0.848
RED: STANDARD DEVIATION=0.720

CHAOS PREDICTION IS 15.1% BETTER THAN FTM

green=FTM

red=CHAOS

white=OBSERVATIONS

Figure 3. FTM and CPM Fits to the Observations for the First Quarter of 1997 (59 Observations)

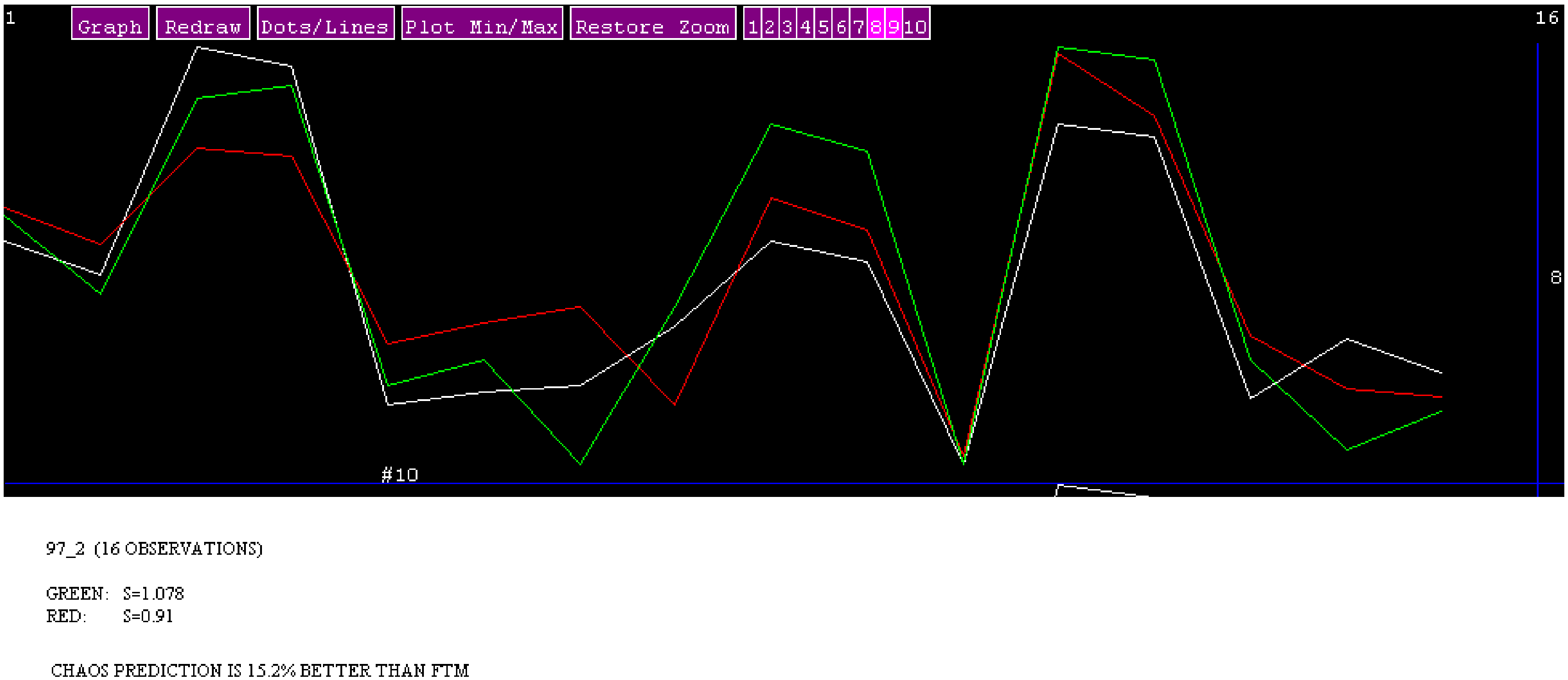


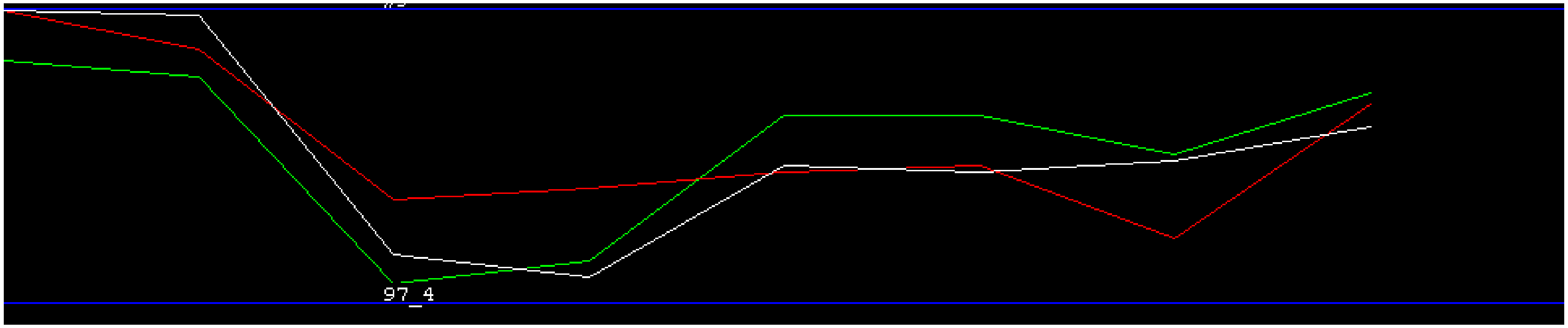
Figure 4. FTM and CPM Fits to the Observations for the Second Quarter of 1997 (16 Observations)

97_3 (10 OBSERVATIONS)

GREEN: S=1.647
RED: S=1.001

CHAOS PREDICTION IS 39.2% BETTER THAN FTM

Figure 5. FTM and CPM Fits to the Observations for the Third Quarter of 1997 (10 Observations)



97_4 (8 OBSERVATIONS)

GREEN: S=0.870
RED: S=0.664

CHAOS PREDICTION IS 23.7% BETTER THAN FTM

Figure 6. FTM and CPM Fits to the Observations for the Fourth Quarter of 1997 (8 Observations)

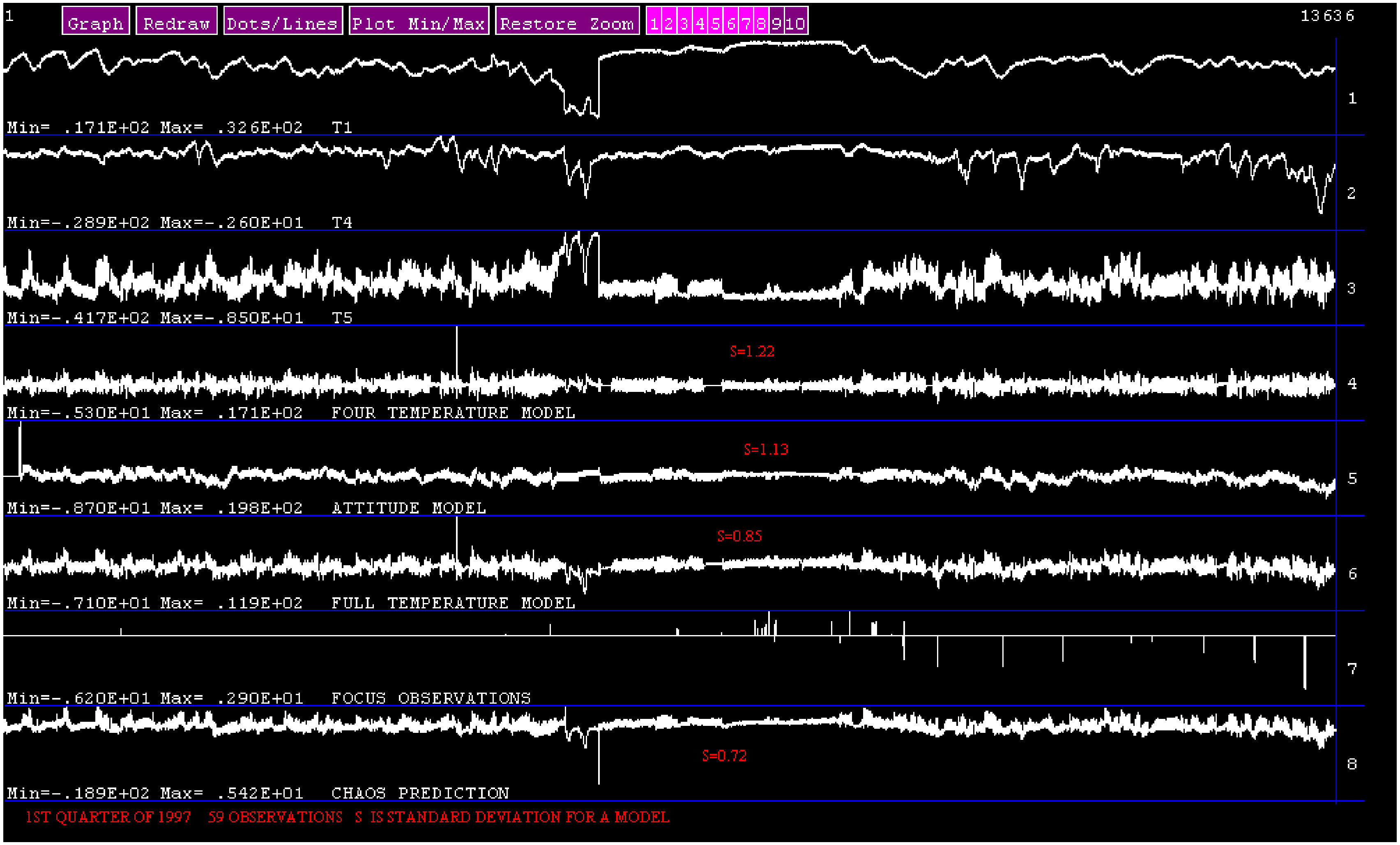
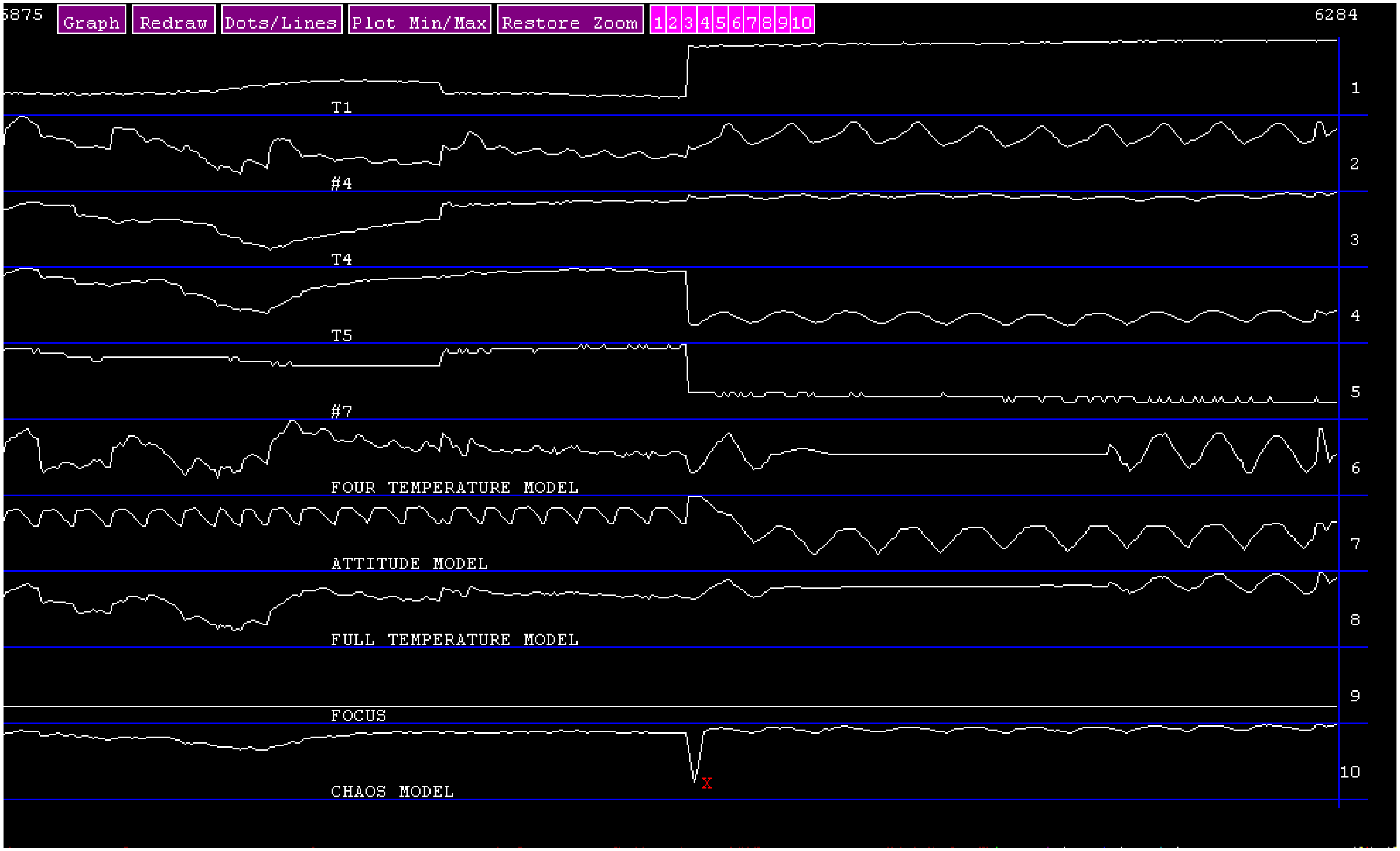
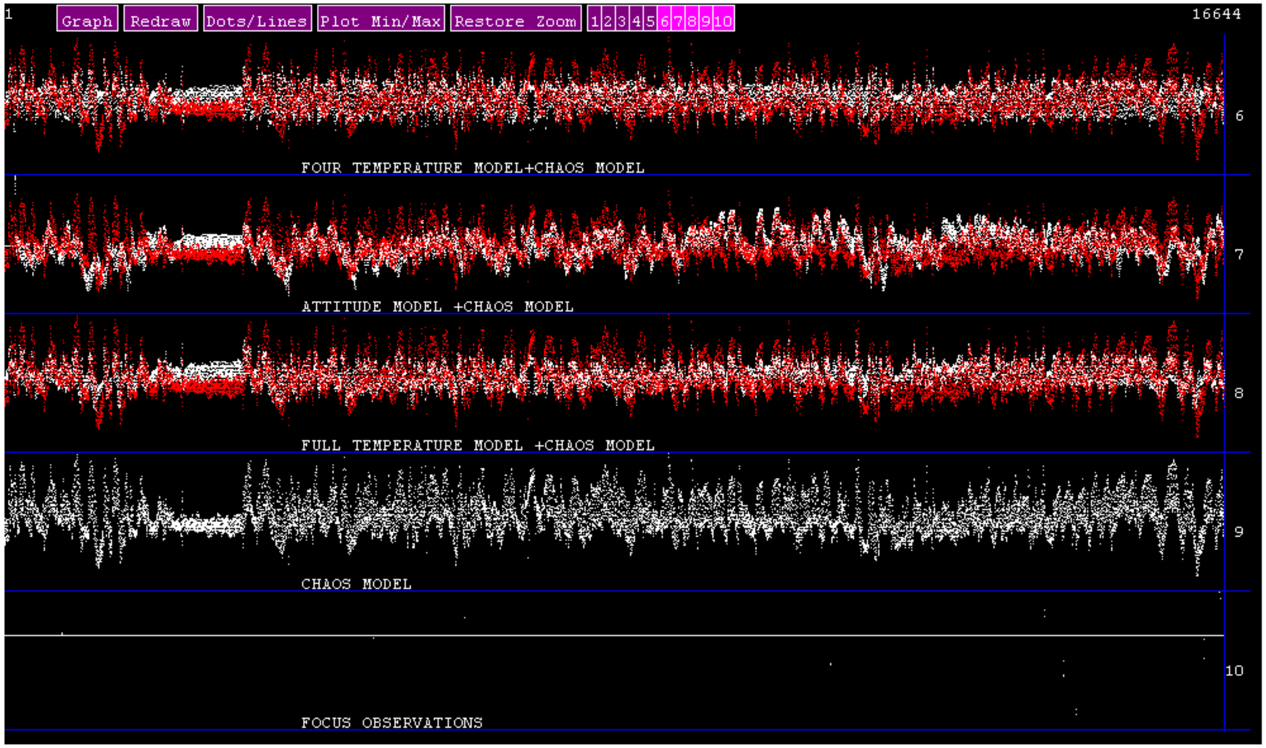


Figure 7a. Overview of Focus Model Behavior for First Quarter of 1997



ZOOM ON THE REGION WHERE GHAOS PREDICTS A SEPARATE DROP X

Figure 7b. Fragment Extracted From Figure 7a for Time Interval of About 25-30 Hours



98_2 QUARTER 18 OBSERVATIONS

S IS STANDARD DEVIATION

FOUR TEMPERATURE MODEL: S=1.917

ATTITUDE MODEL: S=2.765

FULL TEMPERATURE MODEL: S=1.967

CHAOS MODEL: S=0.940

FOCUS PREDICTIONS BY DIFFERENT MODELS

5-MINUTE GRID

WHITE SOLID STRAIGHT LINE IS ZERO LINE IN THE LOW PANEL

DOTS REPRESENT FOCUS VALUES.
THEY ARE OBSERVATIONS ON THE LOW PANEL, AND THEY ARE PREDICTIONS OF DIFFERENT MODELS ON APPROPRIATE PANELS

Figure 8. Comparison of Focus Model Behavior for Second Quarter of 1998 Illustrating Large CPM Amplitudes

Recall that only 184 observed focus measurements were available to construct the focus models. Fortunately, this is sufficient to conduct a statistical test to check inconsistencies in estimated focus amplitudes for the different models. One can plot a histogram of the number of measured foci in an interval of focus values vs. the focus value. Assuming that the focus measurement is a random process (in time) and that the typical time interval of focus variation is much smaller than the time between focus measurements, such a histogram, calculated on the set of measured points only, would be similar to the histogram calculated if exact focus values were known for all times. The model whose histogram is closest to the measured focus histogram should be the more statistically acceptable model.

Figures 9 and 10 show the histograms. Figure 9 gives frequencies (absolute numbers) of amplitudes predicted by the CPM and the FTM that can be compared with the frequencies of amplitudes found for focus observations. This histogram takes into account all 184 focus measurements and the corresponding estimates from the CPM and the FTM. A comparison of the statistics shows that the CPM estimates are closer than the FTM estimates to the observed frequencies.

One can see that the FTM overestimates small amplitudes and underestimates large ones. As a result, when time profiles of focus estimates are overlapped, it appears (but only appears) as though the CPM is overestimating large amplitudes (see Figure 8).

Figure 10 is similar to Figure 9. The only difference is that the estimates of focus amplitude from the CPM and the FTM are for all points (i.e., for all points on the 5-minute time grid, not just the 184 observation points). This figure confirms that the CPM is closer than the FTM to the observed focus distribution in the region of large amplitudes.

CONCLUSION

The CPM fits the focus observations somewhat better than existing physical models do in terms of both achieved LSM deviation on the measured points and the histogram statistics. We expect that the fit could be improved even more if the additional available data (i.e., the four light shield temperatures and the attitude parameters) are taken into account.

We expect also that the CPM could be successfully used for the calibration of other HST/NGST science instruments and devices. The major advantage of the CPM approach is in the fact that it uses the embedding technique in addition to the LSM employed by most calibration software.

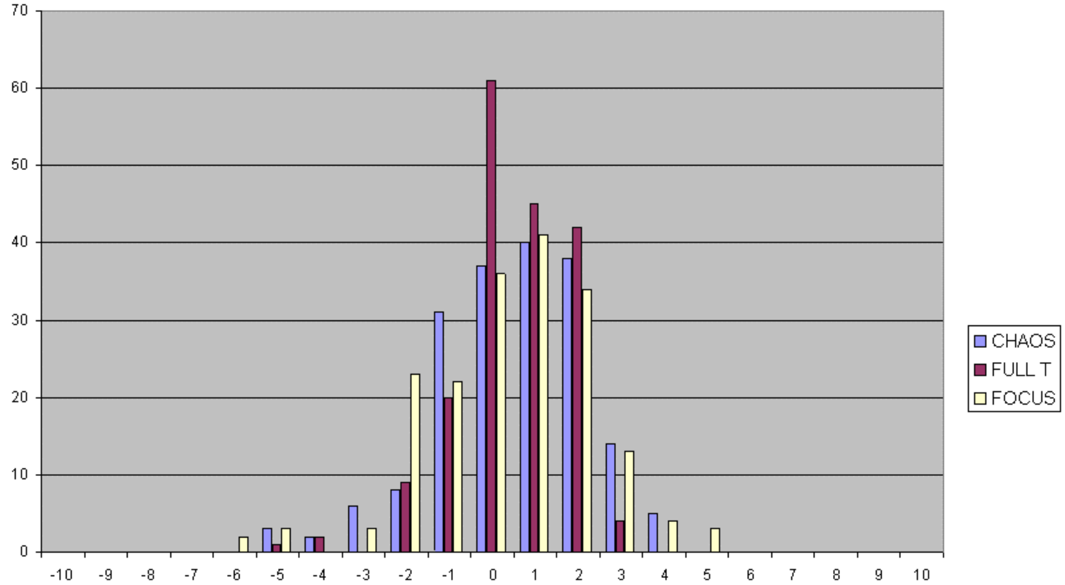


Figure 9. Frequencies of Amplitudes (in Microns) Predicted by CPM and FTM in Comparison With Frequency of Amplitudes Found for Focus Observations (Observational Points Only)

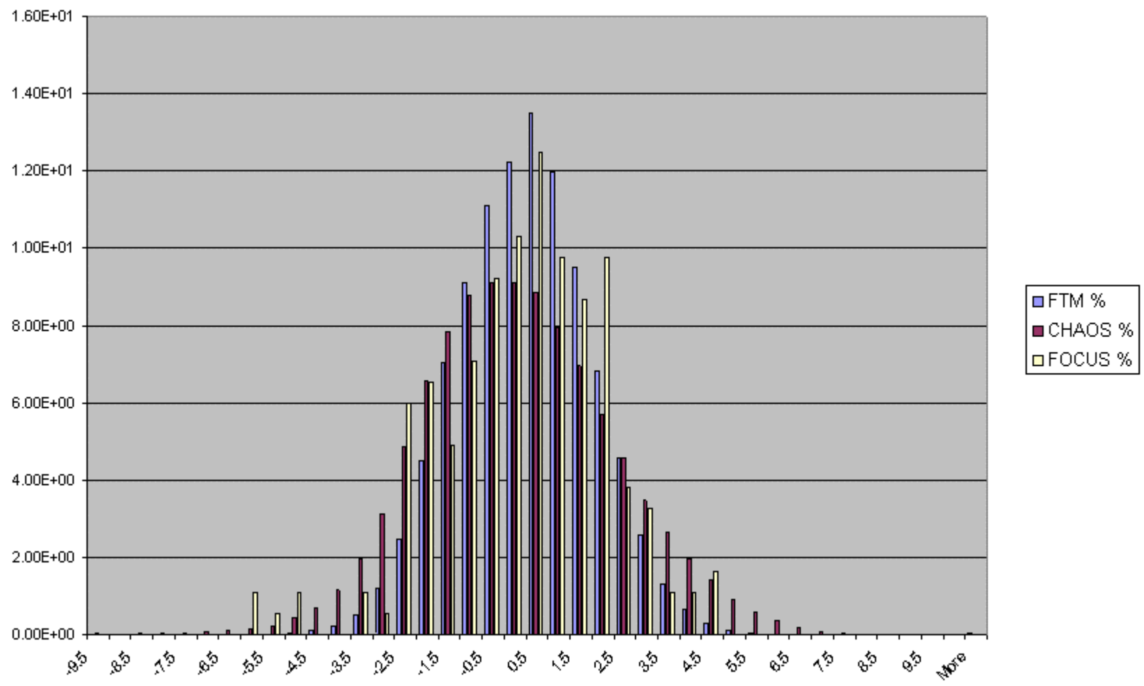


Figure 10. Frequencies of Amplitudes (in Microns) Predicted by CPM and FTM in Comparison With Frequency of Amplitudes Found for Focus Observations (All Points With 5-Minute Grid)

ACKNOWLEDGMENTS

We greatly appreciate the efforts of John Hershey, who provided us with the raw data files, created an interpolator to merge our focus model with his model files (which had different time grids), and commented on our results.

Two of us (LM and RM) wish to thank Mary Galloway and Charlie Wu for providing the opportunity to complete this work efficiently within the framework of Computer Sciences Corporation resources.